Strictly Capturing Non-strict Closures

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Higher-order Functions

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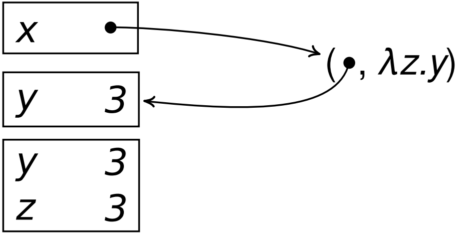

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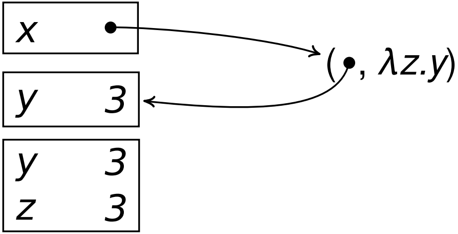

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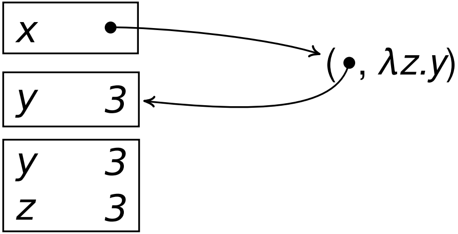

Pairs of environment and code are **closures**.
Here, they are a feature of the runtime system.

In Compilation

What if our compiler target language does not automatically create closures?

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Solution: make closures explicit in the syntax

Closure-conversion

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But what about Haskell?

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  - call-by-name
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  Strictness is an essential aspect of **useful** closure-conversion.

- We propose *partial closure-conversion*, which allows closures to be introduced locally instead of as a total transformation.

Closures in Strict Languages

Closures in Strict Evaluation

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How can we run this program?

Semantics of Target Language

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Useful Closure-conversion

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After closure-conversion, the program does not need a runtime that automatically creates closures.

Closures in Non-strict Languages

More closures for non-strict languages

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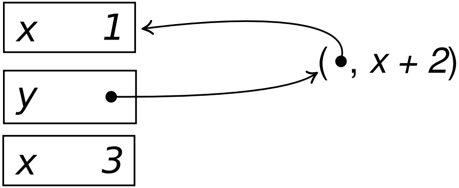

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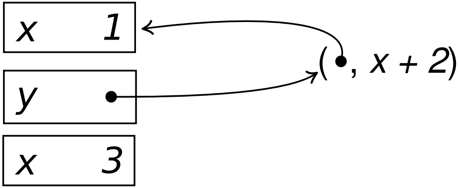

Non-strict languages create thunk closures *in addition to* function closures.

Closures in Call-by-name Languages

Closures in Call-by-name Evaluation

$$\texttt{let}\ x = 1\ \texttt{in}\ \big(\texttt{let}\ y = x + 2\ \texttt{in}\ \big(\texttt{let}\ x = 3\ \texttt{in}\ y\big)\big)$$

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Capturing a thunk closure:

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How can we run this program?

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How can we run this program?

The natural choice is a call-by-name language with data.

Target Language for Call-by-name Closure-conversion

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Non-strict data types do not remove the need for closures in our runtime.

Neither do non-strict functions, nor let-expressions

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What if we simply remove the closure constructing aspect of non-strict data?

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Using non-strict data without a closure constructing target language is wrong.

Target Language for Call-by-name Closure-conversion

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We didn't have this problem for call-by-value closure-conversion.

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Call-by-value data types do not require closures!

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Closures in Call-by-need Languages

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$$\begin{array}{rcl} \texttt{store}\ M & \stackrel{\text{def}}{=} & \texttt{new}\ (\texttt{inr}\ M) \\ \texttt{memo}\ x & \stackrel{\text{def}}{=} & \texttt{case}\ !x\ \texttt{of} \\ & & \quad \texttt{inl}\ v \rightarrow v \\ & & \quad \texttt{inr}\ p \rightarrow \\ & & \quad\quad \texttt{unpack}\ p\ \texttt{as}\ (e, f)\ \texttt{in} \\ & & \quad\quad\quad \texttt{let}\ v = f\ e\ \texttt{in} \\ & & \quad\quad\quad\quad \texttt{let}\ \_ = (x := \texttt{inl}\ v)\ \texttt{in}\ v \end{array}$$

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Is this sufficient for a call-by-need language?

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Such a notion of future heaps applies to a more general notion of memoization with an explicit heap.

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We wish to include the lower-level notion of closures in our compiler's intermediate language **which is lazy**.

Capturing Closures Locally

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This is the same idea as how strict unboxed types are introduced, in Haskell compiler's core.

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